Leaks: quantum, classical, intermediate and more

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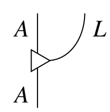
The following is an extended abstract of the paper [John H. Selby and Bob Coecke, *Leaks: quantum, classical, intermediate and more*, Entropy 19.4 (2017): 174.].

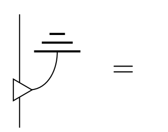
Can we explain why the world is quantum by finding some sense in which quantum theory is an optimal theory? Broadcasting distinguishes quantum theory from classical theory in that quantum states cannot be broadcast [3], but neither can the states of many other theories [2, 5]. Non-locality is a measure of non-classicality, and quantum theory is non-local, but not maximally so [12]. Therefore, is there some manner in which we can uniquely single out quantum theory? In this paper, we show that quantum theory is a leak-free theory, whilst classical theory is maximally leaking. We formalise the notion of a leak, which can roughly be thought of as a 'one-sided broadcasting map', within the process-theoretic framework [1, 5, 6] as a particular type of process, which, as the name suggests, accounts for leaking state-data into the environment. More precisely we define a leak as:

Definition 1. *A leak is a process:*



(1)

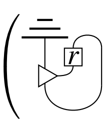
which has discarding [6] as a right counit, that is:



(2)

In theories with feed-back-loops (such as quantum and classical theory where they are provided by compact structure) we can define the *quality* of a leak:

$$\mathsf{Max}_r \left[\frac{1}{D-1} \left(\quad - 1 \right) \right]$$



(3)

where r is a causal process and D is the value of the feedback-loop applied to the identity. It is then simple to see that quantum theory is minimally leaking, as all leaks are constant (i.e. are just the identity transformation composed in parallel with a state preparation) which gives a quality of zero, whilst classical theory is maximally leaking as the broadcasting map provides a leak with quality one.

Next we demonstrate that there is a natural way to introduce leaks to any theory, and by doing so, we obtain new theories. We call this the leak construction. In particular, classical theory can be obtained from quantum theory in this manner, where, in this example, the leaking is then nothing but decoherence [10, 15]. Hence, the concept of a leak allows us to generalise decoherence to arbitrary process theories.

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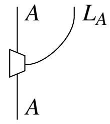
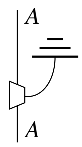
Theorem 2. *Given any process theory and for each system a causal process:*



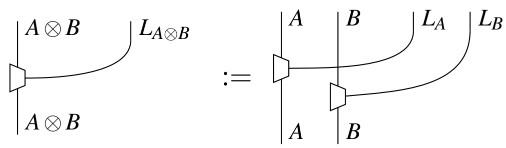
(4)

which is such that the following process is idempotent:



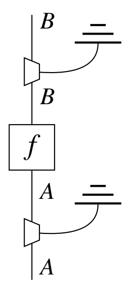
(5)

and which are chosen coherently for composite systems:



(6)

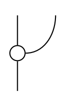
we can construct a new process theory in which each process (4) is a leak for the system A. This construction goes as follows: i) systems stay the same; ii) one restricts processes to those of the form:



(7)

The processes (4), to which we refer in the old theory as pre-leaks, then become leaks in the new theory. Hence, the leak construction turns pre-leaks into leaks. The main motivating example for this construction is of course the following:

Example 3 (Decoherence)**.** *The leak construction for the pre-leak:*



$$: \mathscr{B}(\mathscr{H}) \to \mathscr{B}(\mathscr{H} \otimes \mathscr{H}) :: |i\rangle\langle i| \mapsto |i\rangle\langle i| \otimes |i\rangle\langle i|$$

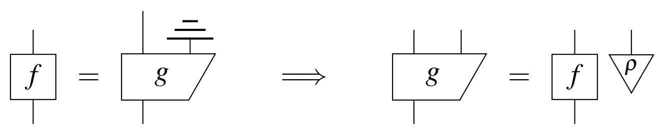
applied to the process theory of quantum processes, we obtain classical probability theory.

Example 3 leaves open the question whether there are any theories that can be obtained from this leak construction in between classical and quantum theory. This question is solved in [7] where the key result is the following theorem:

Theorem 4. *The leak construction applied to quantum processes gives all C*-algebras and C*-algebras only.*

Therefore, despite the weak structure of a leak, for the specific case of quantum theory, we obtain precisely the C*-algebras via the leak construction. Leaks therefore capture the operational content of finite-dimensional C*-algebras on-the-nose, in a manner that does not involve any additive structure, nor a $*$-operation. As explained in detail in the follow-up paper [7], the leak construction is related to the "constructions of classical system types" in [14, 9, 8]. More specifically, in the case of quantum theory, we exactly obtain the same result, but in a much simpler way, with much less use of structure and guided by a clear operational meaning. The notion of a leak is closely related to the decomposability of a state-space [4] in the generalised probabilistic theory framework as, at least under some standard assumptions, such as the "no-restriction hypothesis", each is equivalent to the existence of a non-disturbing measurement.

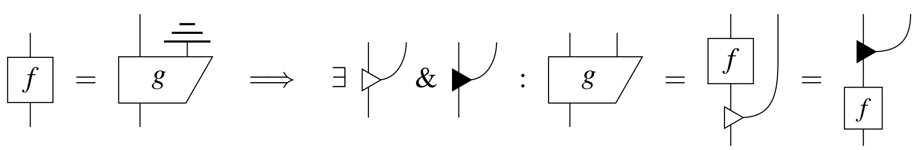
Finally, we observe that defining purity of processes in process theories with leaks is problematic; in particular, this is the case for classical theory. For example, a proposed definition of purity is as follows, f is pure if and only if:



(8)

However, in any theory with non-constant leaks (such as classical theory) this implies that reversible transformations (such as the identity) are not pure. Hence, in such theories we require a more refined notion of process-purity, we propose the following:

Definition 5. *f is pure if and only if:*



(9)

This can be thought of as two conditions, firstly, that any dilation of a pure process is due to system-leakage, and secondly, that the information leaked by the systems does not interact with the pure process. In quantum theory this reduces to the standard notion of purity as all leaks are constant, whilst in classical theory the pure processes are those with at most a single non-zero value in each row and column. In particular, pure causal classical processes are isometries. We also characterise pure processes for composite quantum-classical systems, in particular demonstrating that there is no pure way to transform between quantum and classical information.

In this paper, we have shown how classical theory emerges from quantum theory due to the leak construction, providing a process-theoretic perspective on why the world on a large scale appears to us to be classical. It is natural to ask: Is there some deeper theory of nature than quantum theory from which quantum theory emerges in an analogous way? This is the subject of [11]. A second, related question, would be to ask: What does it imply about a theory if it can obtain classical theory via a leak construction; is the ability for this to happen in quantum theory special or is this a generic feature of general theories? We have also shown that quantum theory is minimally leaking and classical theory maximally; moreover, if we start from a process theory describing finite dimensional C*-algebras, then quantum theory is singled out as the unique minimally-leaking theory. Can this idea lead to a complete reconstruction of quantum theory [13]?

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