Reality of the quantum state: Towards a stronger ψ-ontology theorem

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This note summarises the results in:

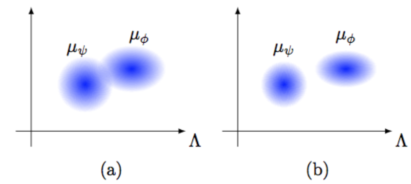
- Shane Mansfield (2016) *Reality of the quantum state: Towards a stronger ψ-ontology theorem.* Physical Review A 94 (4), p.042124. https://doi.org/10.1103/PhysRevA.94.042124

Abstract. The Pusey-Barrett-Rudolph no-go theorem provides an argument for the reality of the quantum state by ruling out ψ-epistemic ontological theories, in which the quantum state is of a statistical nature. It applies under an assumption of preparation independence, the validity of which has been subject to debate. We propose two plausible and less restrictive alternatives: a weaker notion allowing for classical correlations, and an even weaker, physically motivated notion of independence, which merely prohibits the possibility of super-luminal causal influences in the preparation process. The latter is a minimal requirement for enabling a reasonable treatment of subsystems in any theory. It is demonstrated by means of an explicit ψ-epistemic ontological model that the argument of PBR becomes invalid under the alternative notions of independence. As an intermediate step, we recover a result which is valid in the presence of classical correlations. Finally, we obtain a theorem which holds under the minimal requirement, approximating the result of PBR. For this, we consider experiments involving randomly sampled preparations and derive bounds on the degree of ψ epistemicity that is consistent with the quantum-mechanical predictions. The approximation is exact in the limit as the sample space of preparations becomes infinite.

Introduction. A number of recent theorems have addressed the issue of the nature of the quantum state [18, 3, 4, 9, 17]. If we suppose that each system has a certain real-world configuration or state of the matter, the description of which we will call its *ontic state*, then we may pose the question of how a system's quantum state relates to its ontic state. On the one hand, one might consider a pure quantum state as corresponding directly to, or being uniquely determined by, the ontic state, just as the state in classical mechanics (a point in classical phase space) is completely determined by the ontic state or real-world description of the system. On the other hand, the quantum state differs from a classical state of this kind in that in general it may only be used to make probabilistic predictions about the system. Therefore, one might consider that it merely represents our probabilistic, partial knowledge of the ontic state of a system, and moreover that a given ontic state may be compatible with distinct quantum states. Rather convincing plausibility arguments can be made to support either of these views, which are referred to as *ψ-ontic* and *ψ-epistemic*, respectively [10]. The theorems go a step further and prove that under certain assumptions the ψ-epistemic view is untenable. We are especially concerned with the first of these results, the Pusey-Barrett-Rudolph (PBR) theorem [18], which has received the most attention and is considered by some to provide the most convincing case for the reality of the quantum state [12].

Preliminary Report. Final version to appear in:
QPL 2017

Most of the PBR assumptions are common to the familiar no-go theorems of Bell [1], Kochen & Specker [11], etc. These we refer to as ontological assumptions [1]. In addition to the common ontological assumptions, each no-go theorem postulates some form of independence for composite systems or observations; for example, for Bell this is the locality assumption, while for PBR it is the novel assumption of preparation independence. We give a precise definition of preparation independence and provide our own critique of its strength, building on earlier work by the author [14, 13]. In particular, it is pointed out that the assumption is too strong even to allow for classical correlations in multipartite scenarios.



The existence or non-existence of non-trivial epistemic overlaps for distributions induced by pairs of distinct, pure quantum states characterises (a) *ψ-epistemic* and (b) *ψ-ontic* ontological theories, respectively. The epistemic overlap is precisely defined in terms of trace distance as $\omega(\psi,\phi) := 1 - D(\mu_\psi,\mu_\phi)$.

To address the issue, we propose two plausible and less restrictive alternatives. The first allows for classical correlations mediated through a common past, while the second is a much weaker, physically motivated notion of independence, which, as we will explain, is a minimal requirement for enabling a reasonable treatment of subsystems in any theory. We refer to the latter as the *subsystem condition*. The PBR argument is no longer valid under the weaker notions of independence, and this can be demonstrated by means of an explicit ψ-epistemic toy model. At first, this would appear to re-open the door to the possibility of plausible statistical or ψ-epistemic interpretations of the quantum state.

However, we recover two ψ-ontology results which hold under the weaker notions of independence. An intermediate step is to obtain a result which holds in the presence of classical correlations. Our main theorem holds under the minimal notion of independence and approximates the result of PBR. The analysis relies on a proposed experiment involving randomly sampled preparations. The proof makes use of a finite de Finetti theorem [2], establishing a mathematical connection to ψ-ontology results. It also supposes that a certain symmetry present at the phenomenological level is reflected at the ontological level. The theorem places bounds on the degree of ψ epistemicity that is consistent with the experimentally testable quantum-mechanical predictions. Our conclusion of approximate ψ ontology improves monotonically as the size of the sample space of preparations increases and is exact in the limit as the sample space becomes infinite.

Main result The empirical property of quantum theory from which the PBR theorem follows is the existence of *conclusive exclusion measurements* on jointly prepared states. In the simplest case, in which only two subsystems are considered, a conclusive exclusion measurement would give outcomes in the set $\{\neg(\psi,\psi),\neg(\psi,\phi),\neg(\phi,\psi),\neg(\phi,\phi)\}$, with the property that outcome $\neg(\psi,\psi)$ has probability zero whenever the joint preparation (ψ,ψ) has been made, so that its occurrence precludes the possibility of the joint preparation having been (ψ,ψ), and so on.

Theorem 1. *Suppose there exists an m-partite $\{\psi,\phi\}$ conclusive exclusion measurement, and suppose, moreover, that the m subsystems on which the conclusive exclusion measurement is to be performed are uniformly sampled from a larger ensemble of $n > m$ such subsystems. For any symmetric ontological theory satisfying the subsystem condition and describing this experiment, the epistemic overlap of ψ and*

[1] These are briefly summarised in Section 2 of the full article, and the relationship between ψ-ontology and nonlocality/contextuality is dealt with in the appendix.

ϕ is subject to the bound

$$\omega(\psi,\phi) \leq \min\left(\frac{4m|\Lambda|^2}{n}, \frac{2m(m-1)}{n}\right). \tag{1}$$

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