A no-go theorem for theories that decohere to quantum mechanics

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The following is an extended abstract of the paper [Ciarán M. Lee & John H. Selby, *A no-go theorem for theories that decohere to quantum mechanics*, arXiv:1701.07449].

In 1903 Michelson wrote *"The more important fundamental laws and facts of physical science have all been discovered, and these are so firmly established that the possibility of their ever being supplanted in consequence of new discoveries is exceedingly remote"* [Mic03]. Within two years Einstein had proposed the photoelectric effect [Ein05] and within thirty quantum theory was an established field of scientific research. This new science revolutionised our understanding of the physical world and brought with it a litany of classically counter-intuitive features such as superposition, entanglement, and fundamental uncertainty.

Today, quantum theory is the most accurately tested theory of Nature in the history of science. Yet, just as for Michelson, it may turn out to be the case that quantum theory is only an effective description of our world. There may be some more fundamental theory yet to be discovered that is as radical a departure from quantum theory as quantum was from classical. If such a theory exists, there should be some mechanism by which effects of this theory are suppressed, explaining why quantum theory is a good effective description of Nature. This would be analogous to decoherence, which both suppresses quantum effects and gives rise to the classical world [JZK+13, Zur03]. As such, we call such a mechanism *hyperdecoherence* [41].

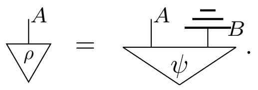
We formalise such a hyperdecoherence mechanism within a broad framework of operationally-defined physical theories by generalising the key features of quantum to classical decoherence. Using this we prove a no-go result: there is no operationally-defined theory that satisfies two natural physical principles, *causality* and *purification*, and which reduces to quantum theory via a hyperdecoherence mechanism. Our result can either be viewed as demonstrating that the fundamental theory of Nature is quantum mechanical, or as showing in a rigorous manner that any post-quantum theory must radically depart from a quantum description of the world by abandoning the principle of causality, the principle of purification, or both.

Here, causality is defined as:

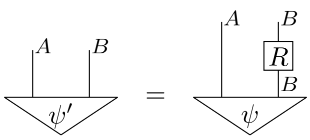
Causality [CDP10]: For each system of type A, there exists a unique deterministic effect, denoted as

This formalises the statement that information propagates from present to future. Purification is defined as:

Purification [CDP10]: For every state on a given system A, there exists a pure bipartite state on some composite system AB, such that the original state arises as a marginalisation of this pure bipartite state:



Here, ψ is said to *purify* ρ. Moreover, any two pure states ψ and ψ' on the same composite system which purify the same state are connected by a reversible transformation



That each state of incomplete information arises in an essentially unique way due to a lack of information about some larger environment system. In a sense, purification can be thought of as a statement of "information conservation";

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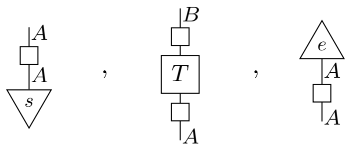
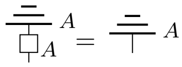
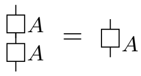
any missing information about the state of a given system can always be accounted for by considering it as part of a larger system.

We formalise the notion of hyperdecoherence as follows. For each system A in the post-quantum theory, there is some process $\square_A$ such that the entirety of quantum theory arises as a sub-theory of the post-quantum theory by appropriately applying $\square$ to states, transformations, and effects from the post-quantum theory. That is, density matrices, completely positive trace non-increasing maps, and POVM elements correspond to



Post-quantum theory: A generalised theory is a post-quantum theory if, for each system type A, there exists a hyperdecoherence map $\square_A$ satisfying the following conditions:

1. $\square_A$ is causal: 
2. $\square_A$ is idempotent: 
3. Pure states in the sub-theory are pure states.

Moreover, the sub-theory defined by the collection $\{\square_A\}$ is quantum theory and at least one of the hyperdecoherence maps must be non-trivial.

We can now state our main result:

Main Theorem. *There is no post-quantum theory satisfying both causality and purification.*

Our result can either be viewed as a justifying why the fundamental theory of Nature is quantum, or as highlighting the ways in which any post-quantum theory must radically depart from a quantum description of the world.

From the famous theorems of Bell [Bel64] and Kochen & Specker [KS67] to more recent results by Colbeck & Renner [CR11], and Pusey, Barrett & Rudolph [PBR12], no-go theorems have a long history in the foundations of quantum theory. Most previous no-go theorems have been concerned with ruling out certain classes of hidden variable models from some set of natural assumptions. Hidden variables—or their contemporary incarnation as ontological models [HS10]—aim to provide quantum theory with an underlying classical description, where non-classical quantum features arise due to the fact that this description is 'hidden' from us.

Unlike these approaches, our result rules out certain classes of operationally-defined physical theories which can supersede quantum theory, yet reduce to it via a suitable process. To the best of our knowledge, our no-go theorem is the first of its kind. This may seem surprising given that it is an obvious question to ask. However, to even begin posing such questions in a rigorous manner requires a consistent way to define operational theories beyond quantum and classical theory. The mathematical underpinnings of such a framework have only recently been developed and investigated in the field of quantum foundations.

As with all no-go theorems, our result is only as strong as the assumptions which underlie it. These will now be critically examined, outlining for each one the sense in which it can be considered 'natural', yet also suggesting ways in which a hypothetical post-quantum theory could violate it, hence escaping the conclusion of our theorem.

Our first assumption is purification. The purification principle provides a way of formalising the natural idea that information can only be discarded [CS15], and any lack of information about the state of a given system arises in an essentially unique way due to a lack of information about some larger environment system. However, proposals for constructing theories in which information can be fundamentally destroyed have been suggested and investigated [OR09, BSP84, UW95]. Such proposals take their inspiration from the Black Hole Information loss paradox. Our result can therefore be thought of as providing another manner in which the fundamental status of information conservation can be challenged.

Our second assumption is causality. This principle allows one to uniquely define a notion of "past" and "future" for a given process in a diagram, and is equivalent to the statement that future measurement choices do not affect current experimental outcomes. As such, this principle appears to be fundamental to the scientific method. Despite this, recent work has shown how one can relax this principle to arrive at a principle of indefinite causality [OCB12, OC14, CDP16, Har07]. In this case, there may be no matter of fact about whether a given process causally precedes another. The indefinite causal order between two processes has even been shown to be a resource which can be exploited to outperform theories satisfying the causality principle in certain information-theoretic tasks [ACB14, Chi12]. Moreover, it has been suggested that any theory of Quantum Gravity must exhibit indefinite causal order [Har16a, Har16b]. Hence, as in the previous paragraph, our result provides further motivation for discarding the notion of definite causal order in the search for theories superseding quantum theory.

As purification seems to require a unique way to marginalise multipartite states, one might wonder whether one can define a notion of purification without the causality principle. Indeed, recent work [AFNB16] has shown how one can formalise a purification principle in the absence of causality, and forthcoming work of one of the authors discusses a 'time-symmetric' notion of purification satisfied by quantum, classical and hybrid quantum-classical systems [SSCng].

Another assumption in our theorem was the manner in which our hyperdecoherence map—the mechanism by which the post-quantum theory reduces to quantum theory—was formalised. It may not be the case that post-quantum physics gives rise to quantum physics via such a mechanism. Indeed, alternate proposals for how some hypothetical post-quantum theory reduces to quantum theory have been proposed [KOSW13]. Despite this, our understanding of the quantum to classical transition in terms of decoherence suggests hyperdecoherence as the natural mechanism by which this should occur.

The last assumption underlying our no-go theorem is the generalised framework itself. While the operational methodology underlying this framework is part and parcel of the scientific method, it may not be the case that the correct way to formalise this methodology is by asserting that pieces of laboratory equipment can be composed together to result in experiments. Indeed, it may be the case that the standard manner in which elements of a theory are composed together—resulting in other elements—needs to be revised in order to go beyond the quantum formalism. Work in this direction has already begun [Har13].

To conclude, our main result can either be viewed as demonstrating that the fundamental theory of Nature is quantum mechanical, or as showing in a rigorous manner that any post-quantum theory must radically depart from a quantum description of the world by abandoning the principle of causality, the principle of purification, or both.

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