Ruling out higher-order interference from purity principles

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The following is an extended abstract of the paper [H. Barnum, C.M. Lee, C.M. Scandolo & J.H. Selby *Ruling out higher-order interference from purity principles* arXiv:1704.05106]

Described by Feynman as "impossible, *absolutely* impossible, to explain in any classical way" [20, volume 1, chapter 37], quantum interference is a distinctive signature of non-classicality. However, as first noted by Rafael Sorkin [39, 40], there is a limit to this interference; in contrast to the case of two slits, the interference pattern formed in a three slit experiment *can* be written as a linear combination of two and one slit patterns. Sorkin has introduced a hierarchy of mathematically conceivable *higher-order* interference behaviours, where classical theory lies at the first level of this hierarchy and quantum theory theory at the second. Informally, the order in this hierarchy corresponds to the number of slits on which the interference pattern has an irreducible dependence.

Many authors have wondered why quantum interference is limited to the second level of this hierarchy [39, 32, 30, 3, 43, 42, 41, 35, 31, 19, 7]. Does the existence of higher-order interference violate some natural physical principle that we believe should be fundamental [33]? In the current work we show that natural physical principles can be found which limit interference behaviour to second-order, or "quantum-like", interference, but that do not restrict us to the entire quantum formalism.

We work in the framework of general probabilistic theories [5, 22, 8, 9, 21, 1, 4, 18, 34, 15, 27, 29, 28, 6]. This framework is general enough to accommodate essentially arbitrary operational theories, where an operational theory specifies a set of laboratory devices which can be connected together in different ways and assigns probabilities to different experimental outcomes. Investigating how the structural and information-theoretic features of a given theory in this framework depend on different physical principles deepens our physical and intuitive understanding of such features. Indeed, many authors [21, 9, 23, 18, 34] have derived the entire structure of finite-dimensional quantum theory from simple information-theoretic axioms—reminiscent of Einstein's derivation of special relativity from two simple physical principles. So far, ruling out higher-order interference has required thermodynamic arguments. Indeed, by combining the results and axioms of Refs. [13, 26], higher-order interference could be ruled out in

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theories satisfying the combined axioms. In this paper we show that we can prove this in a more direct way from first principles, using only the axioms of Ref. [13].

Many experimental investigations have searched for divergences from quantum theory by looking for higher-order interference [38, 37, 36, 25, 24]. These experiments involved passing a particle through a physical barrier with multiple slits and comparing the interference patterns formed on a screen behind the barrier when different subsets of slits are closed. Given this set-up, one would expect that the physical theory being tested should possess transformations that correspond to the action of blocking certain subsets of slits. Moreover, blocking all but two subsets of slits should not affect states which can pass through either slit. This intuition suggests that these transformations should correspond to projectors.

Many operational probabilistic theories do not possess such a natural mathematical interpretation of multi-slit experiments. That is, many theories do not possess well defined projectors [32]. Here, we show that there exist natural information-theoretic principles that both imply the existence of the projector structure and which rule out third-, and higher-, order interference. The following four principles formalise intuitive ideas about the fundamental role of purity in nature:

Axiom 1 (Causality [8, 10])**.** *The probability that a transformation occurs is independent of the choice of tests performed on its output.*

Axiom 2 (Purity Preservation [11])**.** *Sequential and parallel compositions of pure transformations yield pure transformations.*

Axiom 3 (Pure Sharpness [12])**.** *For every system there exists at least one pure effect occurring with unit probability on some state.*

Axiom 4 (Purification [8, 10])**.** *Every state has a purification. Purifications are essentially unique.*

We call theories satisfying these four principles *sharp theories with purification*. We show that such theories possess a self-dualising inner product and that there exist pure projectors which represent the opening and closing of slits in a multi-slit experiment. Barnum, Müller and Ududec have shown that in any self-dual theory in which such projectors exist for every face, if projectors map pure states to pure states, then there can be at most second-order interference [3, Proposition 29]. The conjunction of our new results and the principle of Purity Preservation implies the conditions of Barnum et al.'s proposition. Hence sharp theories with purification do not exhibit higher-order interference. In fact we prove a stronger result, that the systems in such theories are Euclidean Jordan Algebras which have been studied in quantum foundations [43, 3, 2].

These theories are therefore, at least conceptually, very "close" to quantum theory. Moreover, recent work has shown that sharp theories with purification are close to quantum theory in terms of other physical and information processing features. Indeed, such theories possess quantum-like contextuality behaviour [16, 17], quantum-like computation [30, 31], and quantum-like thermodynamic properties [12, 13, 14]. Note that quantum theory is not the only example of a generalised probabilistic theory satisfying these principles. Hence Causality, Purification, Purity Preservation, and Pure Sharpness do not recover the entire quantum formalism. However, if one were to introduce the Ideal Compression and Local Discriminability principles of the reconstruction of quantum theory due to Chiribella, D'Ariano, and Perinotti [9], one would indeed regain the entire quantum formalism. Indeed, both additional principles are necessary: Local Discriminability to preclude real quantum theory and Ideal Compression to preclude the contrived—yet admissible—example of the theory in which all systems are composites of qubits. Sharp theories with purification thus serve as a fertile test-bed for physics that is conceptually quite close to that predicted by the quantum world, but which may diverge from it in certain small, yet interesting, ways.

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